MULTIMEDIA UNIVERSITY

STUDENT ID NO

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 3, 2015/2016

TMT 1181 – MATHEMATICAL TECHNIQUES II

(All sections / Groups)

3 JUNE 2016
9.00 a.m. – 11.00 a.m.
(2 Hours)

INSTRUCTIONS TO STUDENTS

1. This Question paper consists of **6 pages** only including the cover page with **5 Questions** and an **Appendix**.

2. There are **2 Sections**:
   Section A (3 Questions): Attempt **ALL THREE (3)** questions.
   Section B (2 Questions): Attempt **ONE (1)** question **ONLY**.
   All questions carry equal marks and the distribution of the marks for each question is given.

3. Please write your answers in the Answer Booklet provided, and **start each question on a new page.**

4. Show all steps.

SECTION A

Answer ALL THREE (3) Questions.

Question 1

a. Identify the ***linearity*** and ***order*** of the following differential equations:

i. $5y'''+4y''-16y=0$ **[1 mark]**

ii. $\frac{d^7y}{dx^7}-8y^3\frac{dy^2}{dx^2}-9y=0$ **[1 mark]**

b. A ***first order differential equation*** (DE) is given below:

$$\frac{dy}{dx}=\frac{2x}{4y^3+3y^2}$$

i. Show that the DE is ***separable***. **[1 mark]**

ii. Solve the DE using ***separable method***. **[3 marks]**

c. Given the following DE: $x^2\frac{dy}{dx}+2x^5y=x^4$,

i. rewrite the DE into the form of $\frac{dy}{dx}+p(x)y=r(x)$. Identify $p(x)$ and $r(x)$. **[2 marks]**

ii. find the ***integrating factor***, $u(x)=e^{\int p(x)dx}$, of the DE. **[2 marks]**

Question 2

a. Given that the ***roots of the characteristic equation*** for the following ***differential equation*** (DE):

$$ay''+by'+cy=0$$

are $\lambda_1=5$ and $\lambda_2=-1$,

i. find the values of a, b and c. **[2.5 marks]**

ii. write down the ***general solution***, y, of the DE. **[0.5 mark]**

b. Given the following DE: $y''+y'-6y=2x$,

i. determine whether the DE is ***homogeneous*** or ***non-homogeneous***. If it is non-homogenous, convert it to homogeneous DE. Then solve it to get y_h. **[3 marks]**

ii. find the ***particular solution***, y_p, for the DE using ***undetermined coefficients method***. **[3.5 marks]**

iii. form the ***general solution***, $y=y_h+y_p$. **[0.5 mark]**

Continued.....

Question 3

a. Determine the ***Laplace transform*** for the following functions:

i. $f(t) = 3e^{-3t} + \cos 7t$ **[1.5 marks]**

ii. $f(t) = 5 - \sin 0.5t$ **[1.5 marks]**

b. Given that the DE: $f''(t) - 4f'(t) + 3f(t) = 2$,

i. identify the ***linearity*** and ***order*** of the DE. **[1 mark]**

ii. show that the ***Laplace transform*** of $f(t)$ is $F(s) = \dfrac{2}{s(s-1)(s-3)}$ if $f(0) = 0$ and $f'(0) = 0$. **[3 marks]**

iii. the ***partial fraction*** of $F(s) = \dfrac{2}{s(s-1)(s-3)}$ is given in the form of $\dfrac{2}{3s} - \dfrac{1}{s-1} + \dfrac{1}{3(s-3)}$. Find the ***inverse Laplace transform***, $L^{-1}[F(s)]$. **[3 marks]**

SECTION B

Answer ONE(1) Question ONLY.

Question 4

Given the function $f(x) = \dfrac{1}{x} - \sqrt{x^3}$, $x_0 = 0.2$, $x_1 = 0.3$ and $x_2 = 0.4$,

a. find $f(x_0), f(x_1)$ and $f(x_2)$. **[3 marks]**

b. construct the ***Lagrange coefficient*** $L_0(x)$, $L_1(x)$, and $L_2(x)$. **[3 marks]**

c. find the ***second Lagrange interpolating polynomial*** $P_2(x)$. **[2 marks]**

d. approximate $f(0.25)$. **[2 marks]**

(**Note**: Round all the numbers to four decimal places.)

Continued.....

Question 5

a. Given the following linear system of equations:

$$5x+y-z = 3$$
$$-x-3y-z = 2$$
$$2x-y+4z = 3$$

i. Write the system above in ***matrix notation***. **[1 mark]**

ii. Represent the system above as an ***augmented matrix***. **[1 mark]**

b. Refer to the linear system of equations in (a).

i. In what condition the system in (a) can be solved using ***Gauss-Seidel method***? **[1 mark]**

ii. Does the system need to be rearranged into the form of a diagonally dominant system if it is to be solved using ***Gauss-Seidel method***? Then, solve the system using ***Gauss-Seidel method*** by performing the first **TWO iterations**. Given $x_0 = 0, y_0 = 0$ and $z_0 = 0$. (**Note**: Round all the numbers to four decimal places or leave your answers in fraction.) **[7 marks]**

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APPENDIX

LAPLACE TRANSFORMS OF SOME ELEMENTARY FUNCTIONS

Function, $f(t)$	Laplace transform, $F(s)$
1	$\frac{1}{s}$
t	$\frac{1}{s^2}$
$e^{\alpha t}$	$\frac{1}{s-\alpha}$
$\sin \omega t$	$\frac{\omega}{s^2+\omega^2}$
$\cos \omega t$	$\frac{s}{s^2+\omega^2}$
$t^n e^{-\alpha t}$	$\frac{n!}{(s+\alpha)^{n+1}}$
$f'(t)$	$s \cdot F(s) - f(0)$
$f''(t)$	$s^2 F(s) - s f(0) - f'(0)$

General solution of the homogeneous linear second order differential equation

For Distinct real roots: $y = c_1 e^{\lambda_1 x} + c_2 e^{\lambda_2 x}$

For Repeated roots: $y = (c_1 + c_2 x) e^{\lambda x}$

For Complex conjugate roots: $y = e^{\alpha x} (c_1 \cos \beta x + c_2 \sin \beta x)$

Continued.....

Lagrange interpolating polynomial

$$f(x) = \frac{(x-x_1)(x-x_2)...(x-x_n)}{(x_0-x_1)(x_0-x_2)...(x_0-x_n)} f(x_0) + \frac{(x-x_0)(x-x_2)...(x-x_n)}{(x_1-x_0)(x_1-x_2)...(x_1-x_n)} f(x_1)$$
$$+ \frac{(x-x_0)(x-x_1)...(x-x_n)}{(x_2-x_0)(x_2-x_1)...(x_2-x_n)} f(x_2) + ... + \frac{(x-x_0)(x-x_1)...(x-x_{n-1})}{(x_n-x_0)(x_n-x_1)...(x_n-x_{n-1})} f(x_n)$$

Or

$$f(x) = L_0 f(x_0) + L_1 f(x_1) + L_2 f(x_2) + ... + L_n f(x_n)$$

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